Unit 1 First Order Differential Equations

1. The order of a differential equation is order of the largest derivative in it.
2. A DE is linear if it is in the form
3. Separable equations: if the DE can be written as , then the solution is
4. Lost solutions: for the separable DE , all roots of give lost solutions. Specifically, if , then is a solution of . But it will be lost by separation of variables, and we should add the lost solutions.
5. Direction fields: for equation , through each point is drawn a little segment whose slope is

Isoclines: Isoclines are the family of curves constant. All points on a given isocline has the same slope , and thus isoclines can help draw direction fields.

Integral curves: after we sketch the direction fields, we can draw curves which are at each point tangent to the line segment in the direction field. Such curves are called integral curves and are the solution of the DE.

The geometric method for analyzing a DE is: (1) draw isoclines; (2) based on isoclines, draw direction fields; (3) based on direction fields, draw integral curve.

1. Existence and Uniqueness Theorem for ODEs: For any in the region where f is defined and satisfies some technical conditions( is continuous at ), has exactly one solution such that .
2. First order linear equations:

The homogeneous equation is a separable equation and has homogenous solution

Integrating factor: . Then

1. Sinusoidal function:
   * is amplitude
   * is phase lag
   * is time delay or time lag
   * is angular frequency
   * is the frequency
   * is period
2. Autonomous first order differential equations:

* Equilibrium solutions: constant solutions where
* Critical points: where
* Stable equilibrium: an equilibrium solution where all nearby solution curves tend towards it
* Unstable equilibrium: an equilibrium solution where all nearby solution curves tend away from it
* Phase lines: for , draw the y-axis as a vertical line and mark on it the equilibria, i.e. where . In each of the intervals delimited by the equilibria draw an upward pointing arrow if and a downward pointing arrow if .

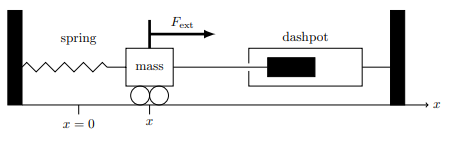
Unit 2 Second Order Constant Coefficient Linear Equations

1. For constant coefficient linear equation the characteristic equation is . If the roots of the characteristic equation are distinct then the general solution is .

If the root repeats times, then are solutions.

When is complex root, we conventionally write as and , since they span the same function space.

1. Damped harmonic oscillator



* Spring force where
* Assume friction is linear damping and only depends on velocity, where
* External force

1. There are three cases for damped harmonic oscillators:
   1. , underdamping. The characteristic equation has complex roots . When the real part , and the solution is simple harmonic oscillator; when , the solution stills oscillates but with decreasing amplitude, as energy is converted to heat.
   2. , overdamping. The characteristic equation has distinct real roots. Notice that both roots are negative, so the solution decays to 0 without oscillate.
   3. , critical damping. Decays to 0 without oscillate. For a fixed and , choosing to be the critical damping value gives the fastest return to its equilibrium position. (As the decay rate is determined by the slowest term.)
2. Superposition I: if are two solutions to a linear homogeneous equation, so is any linear combination

Superposition II: for a linear DE, if is a solution to the homogeneous equation, and is a particular solution to the inhomogeneous equation, then is again a solution to the inhomogeneous equation.

1. Particular solution for different input
   1. Exponential input: then is a particular solution if
   2. Sinusoidal input: